Mathematics-I TU Questions Collection

(B.Sc. CSIT First Semester, TU)

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Chapter 1

Functions of One Variable

1. **[TU 2077, TU 2079, TU 2080, Model Question 2080]**
   Dry air is moving upward. If the ground temperature is 20°C and the temperature at the height of 1 km is 10°C, then

   (a) express the temperature T (in °C) as a function of height h (in km), assuming that a linear model is appropriate.

   (b) Draw a graph for this linear model and find its slope. Also, write the meaning of the slope.

   (c) What is the temperature at the height of 2.5 km?

2. **[TU 2079]**
   If a function f is defined as
   $$f(x) = \begin{cases} 1+x & \text{for } x \leq 1; \\ x^2 & \text{for } x > 1 \end{cases}$$
   then evaluate $f(-3), f(-1), f(0)$ and sketch its graph.

3. **[Model Question 2074]**
   If a function f is defined as
   $$f(x) = \begin{cases} 1-x & \text{for } x \leq 1; \\ x^2 & \text{for } x > 1 \end{cases}$$
   then evaluate $f(-3), f(-1), f(0)$ and sketch its graph.

4. **[TU 2080]**
   Sketch the graph and find domain and range of the function $f(x) = 2x - 1$.

5. **[Model Question 2074]**
   If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, then find $f_\circ g$ and $g_\circ g$.

6. **[Model Question 2080]**
   Write the domain and range of the function $x = y^2$ and sketch its graph.

7. **[TU 2074]**
   If a function f is defined as
   $$f(x) = \begin{cases} x+2 & \text{for } x < 0; \\ 1-x & \text{for } x > 0 \end{cases}$$
   then evaluate $f(3), f(-1)$ and sketch its graph.

8. **[TU 2074]**
   If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$, then find $g_\circ f$ and $g_\circ g$.

9. **[TU 2075]**
If $f(x) = \sqrt{2-x}$ and $g(x) = \sqrt{x}$, then find $f_\circ g$ and $f_\circ f$.

10. **[TU 2075]**
If $f(x) = |x|$, then find $f(-3), f(4)$ and sketch the graph.

11. **[TU 2075]**
Sketch the graph and find the domain of the function $f(x) = x^2 - 6x$.

12. **[TU 2077]**
If $f(x) = x^2$, then find $\dfrac{f(2+h) - f(2)}{h}$.

13. **[TU 2077]**
If $f(x) = \sqrt{x^2 - 1}$ and $g(x) = 2x + 1$, then find $f_\circ g$ and $g_\circ f$. Also, find the domain of $f_\circ g$.

14. **[TU 2078]**
If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$, then find $f_\circ g$, its domain and range.

15. **[TU 2079]**
A rectangular storage container with an open top has a volume of 20m^3. The length of the base is twice its width. Material used for the base costs Rs. 10 per square meter and the material for the sides cost Rs. 4 per meter. Express the cost of materials as a function of the width of the base.

16. **[TU 2078]**
Recent studies have indicated that the average surface temperature of the earth is rising steadily. Scientists have modeled the temperature by the linear function $T = 0.02t + 8.5$ where T is the temperature in centigrade and t represents years since 1900.

    (a) What do slope and T-intercept represent?

    (b) Use the equation to predict average global surface temperature in 2100.

17. **[TU 2073]**
If $f(x) = \sin x$ and $g(x) = -\dfrac{x}{2}$, then find $f\left(f(x)\right)$ and $g\left(f(x)\right)$.

18. **[TU 2071]**
If $f(x) = x + 2$ and $x^3 - 3$, then find $g\left(f(3)\right)$.

19. **[TU 2070]**
Define odd and even functions with an example of each.

20. **[TU 2068]**
Define one-to-one and onto functions with suitable examples.

Chapter 2

Limits and Continuity

1. **[TU 2080]**
   Evaluate the given limit: $\lim_{x\to 0} \dfrac{\sqrt{x^2+9}-3}{x^2}$.

2. **[TU 2080]**
   Show that the function $f(x) = x^2 + \sqrt{7-x}$ is continuous at $x = 4$.

3. **[TU 2077, Model Question 2080]**
   Define continuity at $x = a$. Show that the function $f(x) = \sqrt{1-x^2}$ is continuous on $[-1, 1]$.

4. **[TU 2074, TU 2079, Model Question 2074]**
   Prove that $\lim_{x\to 0} \dfrac{|x|}{x}$ doesn't exist.

5. **[TU 2075]**
   Prove that $\lim_{x\to 2} \dfrac{|x-2|}{x-2}$ doesn't exist.

6. **[TU 2075, Model Question 2074]**
   Define continuity at $x = a$. Show that the function $f(x) = 1 - \sqrt{1-x^2}$ is continuous on the interval $[-1, 1]$.

7. **[TU 2074]**
   Use continuity to evaluate $\lim_{x\to 4} \dfrac{5+\sqrt{x}}{\sqrt{5+x}}$.

8. **[TU 2074]**
   Define limit of a function. Evaluate: $\lim_{x\to\infty} \left(x - \sqrt{x}\right)$.

Chapter 3

Derivatives

1. **[TU 2080]**
   Find y' if $x^3 + y^3 = 6xy$.

2. **[TU 2079]**
   Find the equation of the tangent at $(1, 3)$ to the curve $y = x^2 + 1$.

3. **[TU 2079]**
   State Rolle's Theorem and verify the theorem for $f(x) = x^2 - 9$, $x \in [-3, 3]$.

4. **[TU 2078]**
   Find the equation of the tangent at $(1, 2)$ to the curve $y = 2x^2$.

5. **[TU 2078]**
   State Rolle's Theorem and verify the theorem for $f(x) = x^2 - 3x + 2$, $x \in [0, 3]$.

6. **[TU 2077, Model Question 2074, Model Question 2080]**
   State Rolle's Theorem and verify the theorem for $f(x) = x^3 - x^2 - 6x + 2$, $x \in [0, 3]$.

7. **[TU 2074, TU 2075]**
   Verify Mean Value Theorem for $f(x) = x^3 - 3x + 3$, $x \in [-1, 2]$.

8. **[TU 2074]**
   Find the derivative of $f(x) = \sqrt{x}$ and state the domain of f'.

9. **[TU 2077]**
   Find the equation of the tangent at $(0, 1)$ to the curve $y = x^2 + x + 1$.

10. **[TU 2065, TU 2073]**
    State Rolle's Theorem and verify the theorem for $f(x) = \dfrac{x^3}{3} - x$, $x \in [-3, 3]$.

11. **[TU 2072]**
    State Rolle's Theorem and verify the theorem for $f(x) = x^2$, $x \in [-1, 1]$.

12. **[TU 2071]**
    State Rolle's Theorem and verify the theorem for $f(x) = x^2 - 5x + 7$, $x \in [2, 3]$.

13. **[TU 2071]**
    State Lagrange's Mean Value Theorem and verify it for $f(x) = x^3 - x^2 - 5x + 3$, $x \in [0, 4]$.

14. **[TU 2070]**
    State Rolle's Theorem and verify the theorem for $f(x) = \sin x$, $x \in [0, \pi]$.

15. **[TU 2069]**
    Verify Lagrange's Mean Value Theorem for $f(x) = \sqrt{x(x-1)}$ with $x \in [0, 1]$, if possible.

16. **[TU 2068]**
State Rolle's Theorem and verify that its hypotheses are essential for the theorem to hold using suitable examples.

17. **[TU 2067]**
State Lagrange's Mean Value Theorem and verify it for $f(x) = \sqrt{1 - x^2}$, $x \in [-1, 1]$.

Chapter 4

Application of Derivatives

1. **[TU 2080]**
   Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing.

2. **[TU 2077, Model Question 2080]**
   A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fencing along the river. What are the dimensions of the field that has the largest area?

3. **[TU 2078, Model Question 2080]**
   Find $\sqrt[6]{2}$ correct to eight decimal places using Newton-Raphson Method.

4. **[TU 2079]**
   Sketch the curve of $y = x^2 + 1$ using the guidelines of curve sketching.

5. **[TU 2079]**
   Starting with $x_1 = 1$, find the third approximation, x_3, to the root of the equation $x^3 - x - 5 = 0$.

6. **[TU 2079]**
   Starting with $x_1 = 2$, find the third approximation, x_3, to the root of the equation $x^3 - 2x - 7 = 0$.

7. **[TU 2077]**
   Sketch the curve of $y = \dfrac{1}{x-3}$ using the guidelines of curve sketching.

8. **[TU 2075, Model Question 2074]**
   Starting with $x_1 = 2$, find the third approximation, x_3, to the root of the equation $x^3 - 2x - 5 = 0$.

9. **[TU 2079]**
   Sketch the curve of $y = x^3 + x$ using the guidelines of curve sketching.

10. **[Model Question 2074]**
    Sketch the curve of $y = \dfrac{2x^2}{x^2 - 1}$ using the guidelines of curve sketching.

11. **[TU 2066, TU 2072, TU 2073]**
    Define critical point for a differentiable function. Find critical points for $f(x) = x^2$ and $g(x) = x^{3/2}(x - 4)$.

12. **[TU 2067]**[1]
    Find the points on the curve $xy^2 = 54$ nearest to the origin.

13. **[TU 2065]**
    Sketch the curve of $y = -x^3 + 12x + 5$ using the guidelines of curve sketching within $[-3, 3]$.

[1] For this question, the critical points can be obtained using Lagrange's multipliers too. It is discussed in Unit 10.

14. **[TU 2066]**
Sketch the curve of $y = x^{4/3} - 4x^{1/3}$ using the guidelines of curve sketching.

15. **[TU 2069]**
Sketch the curve of $y = x^{1/2} - 5x^{2/3}$ using the guidelines of curve sketching.

Chapter 5

Antiderivatives

1. **[TU 2080]**
   Evaluate $\int_0^1 3x^2\sqrt{x^3+1}\,dx$.
2. **[TU 2073, TU 2077, Model Question 2080]**
   Show that $\int_1^\infty \frac{1}{x^2}\,dx$ is convergent and $\int_1^\infty \frac{1}{x}\,dx$ is divergent.
3. **[TU 2079]**
   Is the integral $\int_0^3 \frac{1}{x-1}\,dx$ convergent?
4. **[Model Question 2074]**
   Evaluate $\int_0^1 \ln x\,dx$.
5. **[TU 2078, TU 2079, Model Question 2074]**
   Using the method of rectangles, evaluate the exact area under the the parabola $y = x^2$ between $x = 0$ and $x = 1$, bounded below by X-axis.
6. **[TU 2074]**
   Determine whether the integral $\int_1^\infty \frac{1}{x}\,dx$ is convergent or divergent.
7. **[TU 2072]**
   What is an improper integral? Determine whether the integrals $\int_2^\infty \frac{1}{x-1}\,dx$ and $\int_2^\infty \frac{1}{(x-1)^2}\,dx$ are convergent or divergent.
8. **[TU 2067, TU 2068, TU 2070, TU 2071]**
   Determine whether the following integrals are convergent or divergent and evaluate them if possible:
   (a) $\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx$ (b) $\int_2^\infty \frac{1}{(x-1)^{2/3}}\,dx$ (c) $\int_0^1 \frac{1}{1-x}\,dx$ (d) $\int_0^3 \frac{1}{(x-1)^{2/3}}\,dx$
   (e) $\int_{-1}^0 \frac{e^{1/x}}{x^3}\,dx$ (f) $\int_0^\infty \frac{1}{(1+x^2)(1+\arctan x)}\,dx$
9. **[Model Question 2065]**
   Determine whether the integral $\int_{-\infty}^\infty \frac{x^2}{9+x^6}\,dx$ is convergent or divergent. Also, find its value if it is convergent.
10. **[Model Question 2065]**
    Obtain the values of p for which $\int_0^1 x^p \ln x\,dx$ is convergent.

Chapter 6

Applications of Antiderivatives

1. **[TU 2080]**
   Arc of the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$ is rotated about the Y-axis. Obtain the resulting surface area.

2. **[TU 2078, TU 2079]**
   Use trapezoidal rule to approximate $\int_1^2 \frac{1}{x}\, dx$, with $n = 5$.

3. **[TU 2078]**
   Find the area of the region bounded by $y = x^2$ and $y = 2x - x^2$.

4. **[TU 2077, TU 2078, Model Question 2074]**
   Find the volume of the solid formed by rotating the region bounded by $y = x$ and $y = x^2$ about Y-axis.

5. **[TU 2075]**
   Find the volume of the solid formed by rotating the region bounded by $y = x$ and $y = x^2$ about X-axis.

6. **[TU 2075]**
   Find the area of the region bounded by $y = x^2, y = 1, y = 2$.

7. **[TU 2074]**
   Find the area of the region bounded by $x = y^2, x = 0, x = 2$.

8. **[TU 2074]**
   Find the arc length of the semicubical parabola $y^2 = x^3$ between the points $(1, 1)$ and $(4, 8)$.

9. **[TU 2075, Model Question 2074, Model Question 2080]**
   Show that the volume of a sphere of radius a is $\frac{4}{3}\pi a^3$.

10. **[TU 2065]**
    Find the area of the region between the curves $y = \sec^2 x$ and $y = \sin x$ within $\left[0, \frac{\pi}{4}\right]$.

11. **[TU 2065]**
    Find the arc length of the asteroid defined by $x = \cos^3 t,\ y = \sin^3 t$ where $0 \leq t \leq 2\pi$.

12. **[TU 2066]**
    Find the arc length of $y = 2x^{3/2}$ where $0 \leq x \leq 4$.

13. **[TU 2067, TU 2072]**[1]
    Find the area of the region in the first quadrant bounded above by $y = \sqrt{x}$, to the right by $y = x - 2$ and below by X-axis.

[1] It is highly suggested that you use Double Integrals for this question.

14. **[TU 2068, TU 2073]**
    Find the area of the region enclosed by $y = 2 - x^2$ and $y = -x$.

15. **[TU 2069]**
    Find the arc length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$ where $0 \leq x \leq 1$.

16. **[TU 2069]**
    Find the area of the region enclosed by $y = x^2$ and $y = x$.

17. **[TU 2070]**
    Find the area of the region enclosed by $x = 2y^2$, $x = 0$ and $y = 3$.

18. **[TU 2071]**
    Find the area of the region enclosed by $y = \sin x$ and X-axis between 0 and 2π.

Chapter 7

Ordinary Differential Equations

1. **[TU 2080]**
   Show that $y = y(x) = x - \dfrac{1}{x}$ is a solution of the differential equation $xy' + y = 2x$.

2. **[TU 2078, TU 2080]**
   Find the solution to the differential equation $y^2\,dy = x^2\,dx$ that satisfies the initial condition $y(0) = 2$.

3. **[TU 2077, Model Question 2074, Model Question 2080]**
   Find the solution to the differential equation $y'' + y' = 0$ that satisfies the boundary condition $y(0) = 5$ and $y\left(\dfrac{\pi}{4}\right) = 3$.

4. **[TU 2078, TU 2079]**
   A particle moves along a line so that its velocity v at time t is $v(t) = t^2 - t - 6$.

   (a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

   (b) Find the distance traveled during this time period.

5. **[TU 2078, TU 2079]**
   Define initial value problem. Solve: $y'' + y' - 6y = 0; y(0) = 1, y'(0) = 0$.

6. **[TU 2078]**
   Solve: $y' + 2xy - 1 = 0$.

7. **[TU 2077]**
   A particle moves along a straight line and has acceleration given by $a(t) = 6t^2 + 1$. Its initial velocity is 4 and its initial position is 5. Find the position function $x(t)$.

8. **[TU 2075]**
   Define initial value problem. Solve: $y' + 2y = 3; y(0) = 1$.

9. **[TU 2075]**
   Solve: $y'' + 4y' + 4y = 0$.

10. **[TU 2074]**
    Solve: $y'' + 6y' + 9y = 0; y(0) = 2, y'(0) = 1$.

11. **[TU 2074]**
    Define initial value problem. Solve: $y' + 5y = 1; y(0) = 2$.

12. **[Model Question 2074]**
    Define initial value problem. Solve: $x^2y' + xy = 1(x > 0); y(1) = 2$.

13. **[TU 2073]**
    Show that $y = y(x) = ax^2 + b$ is a solution of the differential equation $xy' - y = 0$.

14. **[TU 2073]**
Solve $y'' - y = 0$.

15. **[TU 2072]**
Show that $y = y(x) = ae^x + be^{2x}$ is a solution of the differential equation $y'' + y' - 2y = 0$.

16. **[TU 2072]**
Solve $y'' + y' = 0$.

17. **[TU 2071]**
Show that $y = y(x) = \frac{1}{2}e^x + be^{-x}$ is a solution of the differential equation $y' + y = e^x$.

18. **[TU 2071]**
Solve $y'' + \omega^2 y = 0$.

19. **[TU 2070]**
Show that $y = y(x) = x^2$ is a solution of the differential equation $y' = 2x$.

Chapter 8

Infinite Sequence and Series

1. **[TU 2080]**
   Find the Maclaurin's series expansion of $f(x) = e^x$.

2. **[TU 2075, TU 2080]**
   Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ converges or diverges.

3. **[TU 2066, TU 2067, TU 2075, TU 2077, Model Question 2080]**
   Prove that the Maclaurin's series expansion of $f(x) = \cos x$ converges to $\cos x$ for all $x \in \mathbb{R}$.

4. **[TU 2078, TU 2079]**
   What is sequence? Is the sequence $a_n = \frac{n}{\sqrt{5+n}}$ convergent?

5. **[TU 2079]**
   Find the Taylor's series expansion of $f(x) = \cos x$ about $x = 0$.

6. **[TU 2077, Model Question 2074]**
   Show that the series $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$ is convergent.

7. **[TU 2074]**
   Determine whether the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ converges or diverges.

8. **[TU 2069, Model Question 2074]**
   Prove that the Maclaurin's series expansion of $f(x) = e^x$ converges to e^x for all $x \in \mathbb{R}$.

9. **[TU 2071, TU 2074]**
   Prove that the Maclaurin's series expansion of $f(x) = \sin x$ converges to $\sin x$ for all $x \in \mathbb{R}$.

10. **[TU 2074]**
    Determine the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{x}$ converges.

11. **[TU 2065]**
    Comment on the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 1$.

12. **[TU 2066]**
    Comment on the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

13. **[TU 2067]**

Comment on the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ using integral test.

14. **[TU 2067]**

Comment on the convergence of $\sum_{n=1}^{\infty} \frac{5+2^n}{3^n}$.

15. **[TU 2068]**

Comment on the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ where $p > 1$ using integral test.

16. **[TU 2068]**

Comment on the convergence of $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$.

17. **[TU 2068, TU 2069, TU 2070]**

Test the convergence of following series:

(a) $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ (b) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n!}$ (d) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$.

18. **[TU 2070]**

Prove that the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{5}{4^n}$ is convergent. Also, evaluate its value.

19. **[TU 2071, TU 2072, TU 2073]**

Test the convergence of following sequences:

(a) $t_n = \frac{a - bn^3}{n^3 - c}$ (b) $t_n = \frac{3 - 5n^6}{n^6 - 3}$ (c) $t_n = \frac{a - bn^4}{n^4 + a}$.

Chapter 9

Plane and Space Vectors

1. **[TU 2075, TU 2080]**
   If $\vec{a} = (4, 0, 3)$ and $\vec{b} = (-2, 1, 5)$, then find $|\vec{a}|,\ 3\vec{b},\ \vec{a} + \vec{b},\ 2\vec{a} + 5\vec{b},\ \vec{a} - \vec{b},\ 2\vec{a} + \vec{b}$.

2. **[TU 2079]**
   Find the angle between the lines representing the vectors $\vec{a} = (2, 2, -1)$ and $\vec{b} = (1, 3, 2)$.

3. **[TU 2078]**
   Find a vector perpendicular to the plane containing the points $(1, 4, 6)$, $(-2, 5, -1)$ and $(1, 1, -1)$.

4. **[TU 2078]**
   Find the derivative of $\vec{r}(t) = (1 + t^2)\,\hat{i} + te^t\,\hat{j} + \sin(2t)\,\hat{k}$ and find the unit tangent vector at $t = 0$.

5. **[TU 2077, Model Question 2080]**
   Find a vector perpendicular to the plane containing the points $(1, 4, 6)$, $(-2, 5, -1)$ and $(1, -1, 1)$.

6. **[TU 2077, Model Question 2080]**
   Find the derivative of $\vec{r}(t) = (1 + t^2)\,\hat{i} - te^t\,\hat{j} + \sin(2t)\,\hat{k}$ and find the unit tangent vector at $t = 0$.

7. **[TU 2074]**
   Define cross product of two vectors. If $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} + 7\hat{j} - 5\hat{k}$, find $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

8. **[Model Question 2074]**
   Find the curvature of the circular helix $\vec{r}(t) = (a\cos t, a\sin t, at)$.

9. **[Model Question 2074]**[1]
   Find the shortest distance between the point $(1, 0, -2)$ and the plane $x + 2y + 3z = 4$.

10. **[TU 2073]**
    Find a vector parallel to the line of intersection of $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

11. **[TU 2067, TU 2068, TU 2072, TU 2073]**
    Define curvature of a space curve. Find the curvature of the helix $\vec{r}(t) = (a\cos\omega t, a\sin\omega t, bt)$ with $a > 0, b > 0$ and $a^2 + b^2 \neq 0$.

12. **[TU 2069, TU 2070, TU 2071, TU 2072]**
    Find the angle between

    (a) planes $x - 2y - 2z = 5$ and $5x - 2y - 2z = 0$;

    (b) planes $3x - 6y - 2z = 7$ and $2x + y - 2z = 5$;

    (c) lines representing the vectors $(2, 1, 1)$ and $(-4, 3, 1)$.

[1] This question can also be solved using multivariable calculus.

13. **[TU 2071]**
Define curvature of a space curve. Prove that

(a) curvature of a straight line is zero;

(b) curvature of a circle of radius a is $\frac{1}{a}$.

14. **[TU 2070]**
Find a velocity and acceleration of a particle whose position on the plane at any time t is $\vec{r}(t) = \big(t+1, t^2-1\big)$, when $t = 1$.

15. **[TU 2070]**
Find unit tangent vector, principal normal vector and curvature for the space curve whose vector equation is $\vec{r}(t) = (2\cos t, 3\sin t, t)$.

16. **[TU 2069]**
Define unit tangent vector and principal unit normal vector. Find unit tangent vector and principal unit normal vector for the circle whose vector equation is $\vec{r}(t) = (\cos 2t, \sin 2t)$.

17. **[TU 2068]**
Find the point of intersection of the line $\dfrac{x-(8/3)}{2} = \dfrac{y}{-2} = \dfrac{z-1}{1}$ and the plane $3x+2y+6z = 6$.

18. **[TU 2067]**
Find the equation of the plane passing through $(-3, 0, 7)$ and perpendicular to $\vec{n} = 5\hat{i} + 2\hat{j} - \hat{k}$.

19. **[TU 2066]**
Find the area of the parallelogram with vertices $(0,0)$, $(7,3)$, $(9,8)$ $(2,5)$.

20. **[TU 2066]**
Define unit tangent vector. Find unit tangent vector for the plane curve whose vector equation is $\vec{r}(t) = (\cos t + t\sin t, \sin t - t\cos t)$ with $t > 0$.

21. **[TU 2065]**
Find a vector perpendicular to the plane containing the points $(1,-1,0)$, $(2,1,-1)$ and $(-1,1,2)$.

Chapter 10

Partial Derivatives and Multiple Integrals

1. **[TU 2080, Model Question 2074]**
   If $f(x, y) = x^3 + x^2y^3 - 2y^2$, then find $f_x(2, 1)$ and $f_y(2, 1)$.

2. **[TU 2079]**
   If $f(x, y) = x^2 + x^3y^2 - y^2 + xy$, then find $f_{xx}(2, 1)$ and $f_{yy}(2, 1)$.

3. **[TU 2079]**
   Evaluate $\int_0^3 \int_1^2 x^2y \; dx \, dy$.

4. **[TU 2079]**[1]
   If $z = xy^2 + y^3$ with $x = \sin t$ and $y = \cos t$, then find $\frac{dz}{dt}$ at $t = 0$.

5. **[TU 2079]**
   If $f(x, y) = x^2 + 2x^3y^2 - 3y^2 + x + y$, then find $f_x(1, 2)$ and $f_y(1, 2)$.

6. **[TU 2077]**
   If $f(x, y) = x^2 + 2x^3y^3 - 3y^2 + x + y$, then find $f_x(2, 1)$ and $f_y(2, 1)$.

7. **[Model Question 2080]**
   If $f(x, y) = x^2 + 2x^2y^3 - 3y^2 + x + y$, then find $f_x(2, 1)$ and $f_y(2, 1)$.

8. **[TU 2077, TU 2078, Model Question 2080]**
   If $f(x, y) = x^4 + y^4 - 4xy + 1$, then its local maximum and local minimum values. Also, find its saddle points.

9. **[TU 2077, Model Question 2080]**
   Evaluate $\int_2^3 \int_0^{\pi/2} \left(y + y^2 \cos x\right) \; dx \, dy$.

10. **[TU 2077, Model Question 2074, Model Question 2080]**
    If $f(x, y) = \frac{xy}{x^2 + y^2}$, then does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist?

11. **[Model Question 2074]**[2]
    If $z = x^2y + 3xy^4$ with $x = \sin 2t$ and $y = \cos t$, then find $\frac{dz}{dt}$ at $t = 0$.

12. **[TU 2074, TU 2075]**
    Evaluate $\iint_R f(x, y) \; dA$ where $f(x, y) = 100 - 6x^2y$ and $R := 0 \leq x \leq 2, -1 \leq y \leq 1$.

[1] This question can be solved using ordinary derivatives too.
[2] This question can be solved using ordinary derivatives too.

13. **[TU 2074]**
Find the extreme values of $f(x, y) = y^2 - x^2$.

14. **[TU 2075]**
If $f(x, y) = \dfrac{y}{x}$, then does $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ exist? Justify.

15. **[TU 2075]**
Find extreme values of $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

16. **[TU 2075]**
Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if z is defined as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$.

17. **[Model Question 2065]**
How are Lagrange's multipliers defined? Using these multipliers, find the points on the curve $xy^2 = 54$ that are closest to the origin.

18. **[TU 2065]**
If $f(x, y) = x^2 + 3xy + y - 1$, then find $f_x(2, 1)$ and $f_y(2, 1)$.

19. **[TU 2065, TU 2069]**
Find $\dfrac{dy}{dx}$ using partial derivatives for the following implicitly defined functions:
(a) $x^2 + \cos y - y^2 = 0$ (b) $2xy + \tan y - 4y^2 = 0$

20. **[TU 2066, TU 2070, TU 2071, TU 2072]**
Evaluate the following integrals:
(a) $\displaystyle\int_{\pi}^{2\pi}\int_{0}^{\pi} (\sin x + \cos y)\, dx\, dy$ (b) $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\, dx$ (c) $\displaystyle\int_{1}^{2}\int_{y}^{y^2} dx\, dy$
(d) $\displaystyle\int_{0}^{3}\int_{0}^{2} (4 - y^2)\, dx\, dy$ (e) $\displaystyle\int_{-1}^{0}\int_{-1}^{1} (x + y + 1)\, dx\, dy$

21. **[TU 2066]**
Find u_x if $u(x, y, z) = x^2 + y - z + \sin t$ where $t = x + y$.

22. **[TU 2066]**
Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$, if possible.

23. **[TU 2066, TU 2067, TU 2073]**
Define critical points of a multivariable function $f(x, y)$ defined on a region. Find local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

24. **[TU 2066, TU 2067]**
Find absolute extreme values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular region bounded by the lines $x = 0$, $y = 0$ and $y = 9 - x$.

25. **[TU 2067]**
Evaluate $\displaystyle\iint_R f(x, y)\ dA$ where $f(x, y) = 1 - 6x^2y$ and $R := 0 \le x \le 2, -1 \le y \le 1$.

26. **[TU 2068]**
Find area of the region bounded by $y = x^2$ and $y = x$ using double integrals.

27. **[TU 2068]**
Find the extreme values of $f(x, y) = y^2 + x^2$.

28. **[TU 2068, TU 2069, TU 2072]**
Show that the function

$$f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2} & \text{for } (x,y) \neq (0,0); \\ 0 \quad \text{for } (x,y) = (0,0) \end{cases}$$

is continuous everywhere on the Cartesian plane, except at the origin.

29. **[TU 2068]**
Find maximum and minimum values of $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

30. **[TU 2068]**
State the conditions of the second derivative test for local extreme values. Find local maximum and minimum values of $f(x,y) = x^2 + y^2 + xy + 3x - 3y + 4$.

31. **[TU 2069]**[3]
Let $\phi : \mathbb{R} \longrightarrow \mathbb{R}$ denote a differentiable function. Define a multivariable function $z = z(x,y)$ by $z = z(x,y) = \dfrac{1}{x}\phi(y-x) + \phi'(y-x)$, where $x \neq 0$. Prove that $z_{xx} - z_{yy} = \dfrac{2z}{x^2}$.

32. **[TU 2069]**
Define partial derivatives for $f(x,y)$. Verify mixed partial derivatives theorem for the function $f(x,y) = x^2 + 5xy + \sin x + 7e^x$.

33. **[TU 2069]**
Find local extreme values for $f(x,y) = 2x - 5x^2 - 2y^2 + 4x + 4y - 4$.

34. **[TU 2070]**
If $f(x,y) = x^2 + 2xy + 5$, then find $f_x(1,2)$ and $f_y(1,2)$.

35. **[TU 2070]**
Find local extreme values and saddle points of $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

36. **[TU 2070, TU 2072]**
Find maximum and minimum values of $f(x,y) = xy$ on the ellipse $\dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$.

37. **[TU 2071, TU 2073]**
Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{2y^2}{x^2+xy}$, if possible.

38. **[TU 2071]**
Find local extreme values for $f(x,y) = x^3 + y^3 - 12x + 20$.

39. **[TU 2071]**
Find maximum and minimum values of $f(x,y) = xy$ on the ellipse $x^2 + 2y^2 = 1$.

40. **[TU 2072]**
Find equation to the tangent plane for the surface $z = 9 - x^2 - y^2$ at $(1,2,4)$.

41. **[TU 2072]**
Find maximum and minimum values of $f(x,y)x^3 - y^3 - 2xy + 6$.

42. **[TU 2073]**
Evaluate $\lim\limits_{(x,y)\to(0,1)} \dfrac{x - xy + k}{x^2y + 5xy - y^3}$, if possible.

43. **[TU 2073]**
Find maximum and minimum values of $f(x,y) = xy$ on the circle $x^2 + y^2 = 1$.

[3]The equation that we see in this question is called a **Partial Differential Equation**.